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Collapsing Stellar Cores and Supernovae

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Summary. The evolution of a stellar core is studied during its final quasi-hydrostatic contraction. The core structure and the (poorly known) properties of neutron rich matter are parametrized to include most plausible cases. It is found that the density-temperature trajectory of the material in the central part of the core (the core-center) is insensitive to nearly all reasonable parameter variations. The central density at the onset of the dynamic phase of the collapse (when the core-center begins to fall away from the rest of the star) and the fraction of the emitted neutrinos which are trapped in the collapsing core-center depend quite sensitively on the properties of neutron rich matter. We estimate that the amount of energy E_{cm} which is imparted to the core-mantle by the neutrinos which escape from the imploded core-center can span a large range of values. For plausible choices of nuclear and model parameters E_{cm} can be large enough to yield a supernova event.

Key words: Supernovae — stellar evolution

I. Introduction

Massive stars consume the nuclear fuel in their cores and eventually evolve to the state where they have a central region composed of iron peak elements which is surrounded by successive layers of lighter elements which are still undergoing nuclear burning. Recent calculations have indicated that isolated, slowly rotating stars of masses greater than about 7 $M_\odot$ will develop hot, relativistically degenerate iron cores with masses near the Chandrasekhar limit of 1.4 $M_\odot$ (Arnett, 1973; Barkat, 1977). It is well known that stars with this type of configuration eventually become unstable to core collapse. The combined effects of electron capture reactions, which decrease the electron degeneracy pressure, and high temperature nuclear dissociations which decrease the thermal component of the pressure, rob the core of its support and send it into dynamic infall.

As the stellar core collapses from its initial stable configuration with a central density of the order of 10^9 g cm^{-3} to neutron star densities of 2 10^{14} g cm^{-3} or beyond to a black hole, the gravitational binding energy of the central solar mass or so increases by about 10^{53} erg. Eventually, this change in the gravitational binding energy results in a comparable energy flux of neutrinos. Since the energy content of observed supernova events is deduced to be of the order of 10^{50} erg, it is enticing to imagine that a fraction of a percent or so of the energy emitted in neutrinos is coupled to the matter of the star and thereby powers the supernova. Much work has been done along these lines.

The early work on neutrino transport supernovae was performed prior to the discovery of neutral current weak interactions. At that time the neutrinos were thought to interact most strongly with the electrons. The inclusion of neutral current effects modifies this picture. By means of neutral current interactions, neutrinos scatter off nuclei, and the neutrino–nuclei interaction is considerably greater than neutrino–electron interaction (Lamb and Pethick, 1976). Because the rest energy of the nuclei is far greater than the energy which can be conceivably expected for the supernova neutrinos, a neutrino transfers negligible energy when it scatters off a nucleus which is at rest. Nevertheless a neutrino does impart a large fraction of its momentum to the nucleus. These facts have suggested that supernova may occur by a "neutrino-momentum transport process", in which the neutrinos directly transmit a pulse of momentum to the outer region of the core (the core-mantle). The core-mantle then expands and generates a shock wave (or strengthens a shock which is produced by the rebound of the core center) which propagates into the outer portions of the star. When this shock wave reaches the stellar surface it may produce the observable characteristics of a supernova (Falk and Arnett, 1977; Lasher et al., 1977).

The numerical calculations of collapsing iron-core stars which have been performed to date do not yield a clear or definitive conclusion as to whether the neutrino-momentum-transport mechanism or any other mechanism can produce a supernova explosion (cf. Bruenn, Arnett and Schramm, 1977). This indeterminate status is due to the complexity of the problem: the initial, pre-collapse configuration is not completely understood, and the task of including all of the relevant

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physics and executing the numerical calculations with sufficiently high accuracy has proven to be quite difficult.

In the present investigation we have examined the processes which occur during the collapse of an iron stellar core in order to elucidate the phenomenology of a neutrino-momentum-transport supernova, and to clarify which parts of the input physics and other aspects of the problem require special care in complete numerical investigations.

In Sect. II we examine the early evolutions of the stellar core up until the time at which the core collapse becomes rapid, the onset of dynamic collapse. Prior to this time the inertial terms in the force equations are small and the stellar core essentially has a hydrostatic structure. The structure of the core-mantle is thus directly determined by the core conditions at the center of the core. Since the core-mantle changes little during the collapse of the core-center, its structure during the supernova event is determined by the central conditions (especially the density) at the onset of dynamic collapse. A criterion is derived for the onset of dynamic collapse. Of particular concern is how the conditions at the onset of dynamic collapse depend on the input nuclear physics and on the stellar models. In Sect. III we examine the interactions of escaping neutrinos and estimate how much energy can be transferred to the core-mantle. Section IV contains a summary of our main conclusions.

II. Onset of Dynamical Collapse

Picture the core as being composed of two regions, the core-center (*cc*) and the core-mantle (*cm*). The core-center, which is at the higher density, is capable of collapsing in a time t_{cc} which is short compared to the time which is required for the core-mantle to change appreciably, t_{cm}. However, as long as the core-center is evolving on a time scale similar to t_{cm}, the entire core is in approximate hydrostatic equilibrium and the density structure of the core-mantle is directly related to the conditions at the center of the core.

Eventually the core-center starts evolving fast compared to t_{cm}, falls away from the mantle and collapses at a rate determined by its own dynamic time-scale. The delay from the onset of this dynamic collapse until the time at which the mantle is jarred by the energy released by the core-center collapse (in the form of neutrinos and/or shock waves) is of the order of t_{cc}. Since the core-mantle moves very little during this interval, *the structure of the core-mantle during a supernova explosion is determined by the conditions at the center of the stellar core at the onset of dynamic collapse.*

During the hydrostatic evolution of a highly evolved star, the stellar core is similar to an iron white dwarf; the central pressure and density are largely determined by the mass of the core. As the Si burning shell adds mass to the iron core the central density increases. This is a slow process compared to the hydrodynamic timescale t_{cm}. Meanwhile in the core energy is lost by neutrino and photon radiation and energy is expended as heavy nuclei dissociate into alpha particles and free nucleons. In the early evolution the internal processes (mainly the β-processes) are slow compared to the rate of which the stellar core can relax hydrodynamically. The core thus evolves hydrostatically. Eventually, at sufficiently high densities and temperatures, the energy losses and/or the rate of dissociation become more rapid than the hydrodynamic relaxation rate for the core-mantle and dynamic collapse begins. The evolution of the core-center can thus be conceptually divided into two processes: the inexorable increase in density due to the external effects of the Si burning shell and the internal evolution of the matter due to energy losses and decomposition of the heavier nuclei.

To estimate the point at which dynamic collapse begins we consider the core-center while it is in hydrostatic equilibrium with the core-mantle and examine how the core-center responds to small changes in the core-mantle.

The dynamics of the core-center are governed by the internal pressures and gravitational binding of the core-center and by the external pressure of the overlying core-mantle material. As long as the core center is roughly in hydrostatic equilibrium, i.e. the motions in the core center are very subsonic, the Virial equation gives

$$0 = 3(P_{cc} - P_{cm})V_{cc} - \Omega_G. \tag{1}$$

Here P_{cc} is the average pressure in the core-center, V_{cc} is the volume, P_{cm} is the external pressure due to the core-mantle and Ω_G is the gravitational binding energy of the core-center. The core-center is roughly uniform so that

$$\Omega_G = \frac{3}{5}\frac{GM_{cc}^2}{R_{cc}} \tag{2}$$

is a good approximation.

Substitution of $\rho_{cc} = M_{cc}/V_{cc}$ into the above equations to eliminate V_{cc} and R_{cc} and differentiation with respect to P_{cc} gives

$$\frac{dP_{cc}}{dP_{cm}} = [1 - (M_{cc}/M_{\rm crit})^{2/3}]^{-1}, \tag{3}$$

where

$$M_{\rm crit} \equiv \left(\frac{3}{4\pi\rho_{cc}}\right)^2 \left(\frac{5\pi\gamma P_{cc}}{G}\right)^{3/2} \tag{4}$$

and

$$\gamma \equiv \frac{d(\ln P_{cc})}{d(\ln \rho_{cc})}. \tag{5}$$

For numerical evaluation of $M_{\rm crit}$, take P_{cc} to be θ times the zero temperature Fermi pressure; i.e. $P_{cc} = 1.244\ 10^{15}\ \theta(\rho Y_e)^{4/3}$, where Y_e is the number of electrons per nucleon mass. One then obtains

$$M_{\rm crit} = 4.5(\theta\gamma)^{3/2}\, Y_e^2\, M_\odot. \tag{6}$$

Our calculations show that in almost all cases θ is very near unity in the relevant regimes.

As the core-center evolves under an ever increasing external core-mantle pressure, the internal pressure increases according to Eq. (3). During the evolution the electron number and the value of γ decrease due to electron capture reactions. These effects decrease $M_{\rm crit}$ until it approaches the mass of the core-center. As this limit is approached, the core-center pressure (and therefore the core-center density) changes very rapidly to adjust to slow changes in the core-mantle pressure. Since the core-mantle requires a fairly long time to relax to hydrostatic equilibrium, it cannot readjust to the changes in the core-center and hydrostatic equilibrium is no longer maintained. The condition for the onset of dynamic collapse of the core center is thus $M_{\rm crit} \approx M_{cc}$.

For the core-center masses of the order of 0.5 $M_\odot$, the electron number at the onset of dynamical collapse is found from Eq. 6 to be

$$Y_e = 0.33(\theta\gamma)^{-3/4}(M_{cc}/0.5\,M_\odot)^{1/2}. \tag{7}$$

Early Evolution of the Core

To understand the development of the core prior to the onset of collapse, we examine an element of matter at the center of the core. During the hydrostatic phase of the core evolution, the central density, ρ_c, evolves at the same rate as the core-mantle

$$\frac{1}{\rho_c}\frac{d\rho_c}{dt} = t_{\mathrm{cm}}^{-1}. \tag{8}$$

The hydrostatic evolution becomes more rapid as the onset of dynamic collapse is approached. Just prior to the collapse of the core-center, the mantle is evolving at nearly its maximum rate. The minimum hydrodynamic timescale for the mantle is approximately given by

$$t_{\min} \approx (24\pi G\rho_{\mathrm{cm}})^{-1/2}. \tag{9}$$

This is the free fall timescale for a homogeneous sphere. If the entire iron core is approximated by an $n = 3$ polytype, which is appropriate for a relativistically degenerate object, then the average core density is about 0.018 of the central density. Taking this as an estimate of the typical mantle density gives $t_{\min} \approx 3.3\;10^3\;\rho_c^{-1/2}$ s.

In our calculations we use the following functional form for t_{cm}

$$t_{cm} = \chi t_0(\rho_c/\rho_0)^\beta, \tag{10}$$

where $t_0 = 3300\;\rho_0^{-1/2}$, ρ_0 is the density at which the calculations are started, and χ is a dimensionless parameter. The central density is therefore given by

$$\rho_c = \rho_0\left(1 + \frac{\beta t}{t_0\chi}\right)^{1/\beta}. \tag{11}$$

Calculations have been performed with several values of β. We have found that the choice $\beta = -1$ provides the best agreement with the density evolution that is obtained in detailed hydrodynamic studies of core contraction (Arnett, 1977b; Wilson, 1976; Van Riper, 1978) and we have used this value throughout the present investigation. Our main results are nevertheless more general and are valid as long as the contraction time of the core is of the order of $t_{\min}$.

The ratio of the above timescale for core contraction to the free fall timescale for a homogeneous distribution of matter at density ρ_c is

$$\xi = \frac{t_{cm}}{(24\pi G\rho_c)^{-1/2}} = 7.4\chi(\rho_c/\rho_0)^{(2\beta+1)/2}. \tag{11a}$$

With the values of the collapse parameters which give good fits to the calculations of Arnett (1977b), ($\beta = -1$, $\chi = 18.4$ and $\rho_0 = 4\;10^9\;\mathrm{g\,cm^{-3}}$) the value of ξ goes from 136 at $\rho_c = \rho_0$ to 8.6 at $\rho_0 = 10^{12}\;\mathrm{g\,cm^{-3}}$. If β is less than $-\frac{1}{2}$, ξ decreases with increasing density. Equation (10) can thus yield unreasonably short collapse timescales if extrapolated to higher densities than those for which is is intended.

The evolution of a mass element at the center of the iron core is now determined by the rate of energy loss and by the rate of change of the electron number:

$$\frac{du}{dt} = -p\frac{dV}{dt} - q, \tag{12}$$

$$\frac{dY_e}{dt} = \dot{Y}_e. \tag{13}$$

In these equations u is the specific internal energy, P is the total pressure, $V \equiv 1/\rho_c$, and q is the specific neutrino energy loss rate (photons are assumed to escape at a negligible rate during this stage of the core evolution). For the time being it is assumed that neutrinos are not retained in the core.

Equations (12) and (13) can be integrated if P, q and $\dot{Y}_e$ are specified as functions of u, Y_e and ρ_c. In fact it is more convenient to specify these quantities as function of Y_e, ρ_c and T. If this is done then an additional relation is needed for $T(u, Y_e, \rho)$. For the study of the onset of collapse one is concerned with $\rho_c \lesssim 10^{12}\;\mathrm{g\,cm^{-3}}$. In this regime the internal energy is obtained from

$$u = u_I + u_e + u_{\mathrm{nuc}} + u_{\mathrm{coul}}, \tag{14}$$

where u_I is the thermal energy of the ions, u_e is the electron internal energy, u_{nuc} is the nuclear potential energy (the negative of the nuclear binding energy) and u_{coul} the Coulomb energy of the dense plasma.

The total pressure is similarly

$$P = P_I + P_e + P_{\mathrm{coul}} \tag{15}$$

the evaluation of the terms in Eqs. (12)–(15) is described in the Appendix.

Equation (14) is solved by a Newton-Raphson technique to give $T(\rho, Y_e, u)$ and Eqs. (12) and (13) are numerically integrated to yield the electron number and u as a function of time.

At the completion of core silicon burning, the central temperature of the core is about $4\;10^9$ K, and the density depends on the stellar mass. Specific values of the post-silicon

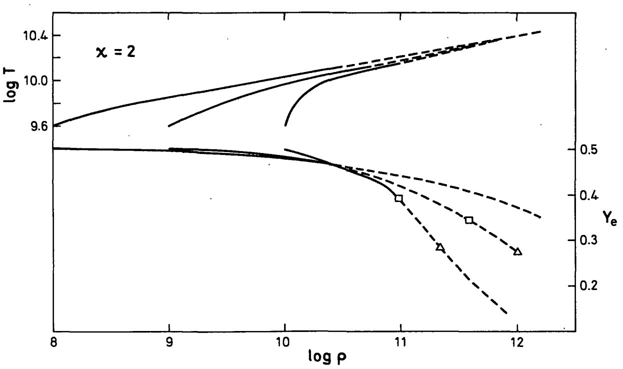

Fig. 1. The evolution of the central temperature and electron number are shown as a function of the central density. The "standard" nuclear parameters are used, and the density evolution parameters are $\chi = 2$ and $\beta = -1$. Note that the collapse rates depend on the initial densities. The curves become dashed in the regime where neutrino trapping is possible (for $M_{cc} = 0.5\,M_\odot$ and $\delta = 1$). The squares and triangles indicate where $M_{\mathrm{crit}}/M_\odot$ equals 0.7 and 0.3, respectively

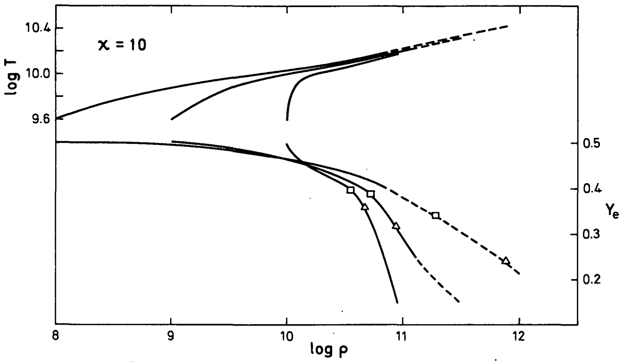

Fig. 2. The same as Fig. 1 with $\chi = 10$ (slower contraction rate)

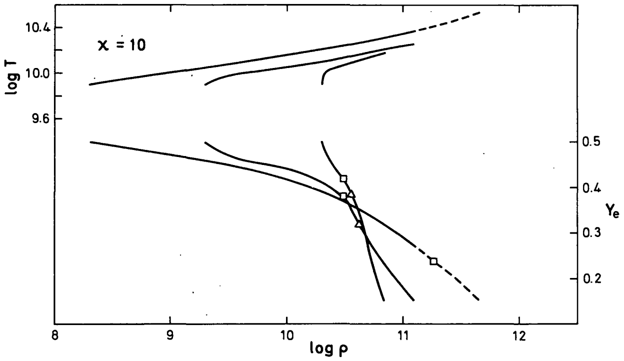

Fig. 4. The same as Fig. 2. (Note that the evolution starts at a higher temperature)

burning central densities are difficult to obtain since the complex evolution through Si burning is not yet well understood (Arnett, 1977a). A reasonable range of estimates would be 10^8–10^{10} g cm^{-3} where the lower densities are for stars greater than $\cong 30\ M_\odot$ and the higher densities correspond to stars of $\lesssim 10\ M_\odot$.

In Figs. 1–3 we show the evolution of the temperature and electron number as a function of density for initial post Si-burning temperature of $4\ 10^9$ K and densities of 10^8, 10^9, 10^{10} g cm^{-3} and values of $\chi = 2$, 10 and 30, respectively. In these figures we have used the "standard" nuclear parameters (see the Appendix). Note that the contraction time at a given density ρ_c is $t_{cm} = \chi(\rho_0/\rho_c)^{1/2}\ t_{\min}$, so that large values of χ do not necessarily indicate very slow contraction rates.

The striking feature of these figures is the rapid convergence to a common trajectory which is independent of the compression rate. This behavior is caused by the heating or cooling effects of electron capture reactions and by the thermal dissociation of nuclei (Nakazawa, 1973). Each electron capture reaction removes one electron and thus lowers the energy of the electron gas by approximately the Fermi energy. This energy can be disposed of in several ways: some of it will be carried off by the escaping neutrino and some of it will be absorbed by the nuclei which are becoming progressively less bound as the core material becomes hotter and more neutron rich. The remaining energy (if there is any) will heat the ions and electrons. If the sum of the neutrino energy and the nuclear contribution exceeds the energy which was made available by the electron capture, then the thermal energy will be decreased as a result of each electron capture. At low temperatures the electron capture reactions tend to heat the matter and at high temperatures the effects of the neutrino emission and the change in the nuclear binding energy tend to cool the matter. Since the electron Fermi energies and the nuclear binding energies in the stellar core material are much greater than the thermal energies, the electron capture and nuclear heating and cooling processes rapidly achieve near equilibrium so that the $\rho - T$ trajectories converge before a significant fraction of the electrons are captured.

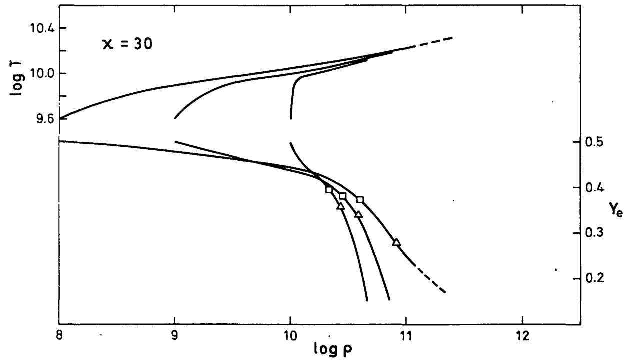

Fig. 3. The same as Fig. 2 with $\chi = 30$ (even slower contraction rate)

If the thermal energy is high relative to the electron energy and the variations in the nuclear binding energy, the neutrino and nuclear cooling processes cannot drive the temperature very far down. In these cases the $\rho - T$ trajectories will not converge. This is illustrated in Fig. 4 where the initial temperature is set to $8\ 10^9$ K, far above the temperature at the conclusion of silicon burning. It is apparent that the highest entropy trajectory does not converge toward the lower ones. Actually high entropy trajectories of this sort are not relevant to the initial collapse of a stellar core. If a core completes consumption of silicon at even moderate densities of greater than 10^8 and temperatures below about $5\ 10^9$ K, its evolution will be similar to those shown in Figs. 1–3. The phenomenon of convergence to a common density-temperature evolution appears to be a universal feature of stellar core contraction.

In the calculations discussed so far, we have assumed free escape of the neutrinos. The escape of the neutrinos from the core is mainly impeded by elastic scattering off nuclei. As long as the diffusion time $t_{\rm diff}$ due to this elastic scattering is short compared to the evolutionary time scale of the core [Eq. (8)], the neutrinos certainly escape. When $t_{\rm diff}$ is longer than this neutrinos are at least partially trapped and tend to accumulate in the core. In this latter case our computations assuming free escape are no longer valid.

The diffusion time for a neutrino emitted in a homogeneous core of radius R and density ρ is typically

$$t_{\rm diff} = \frac{R^2}{c\lambda}, \qquad (16)$$

where the mean free path for elastic scattering of neutrino from nuclei is (Lamb and Pethick, 1976)

$$\lambda = 5\ 10^{16} \varepsilon_{20}^{-2} \left(X_{\rm H} \frac{A_{\rm H}}{60} + 0.2\ X_n \right)^{-1} \rho^{-1}\ \text{cm}. \qquad (17)$$

Here ε_{20} is the neutrino energy in units of 20 MeV, X_n is the mass fraction of neutrons and $X_{\rm H}$ is the mass fraction of heavy nuclei which have a characteristic atomic weight of $A_{\rm H}$.

To obtain an estimate for the neutrino diffusion time, we characterize the core by a single density $\rho = \delta\rho_c$ and specify that the electron capture neutrinos are emitted near the electron Fermi energy, $\varepsilon_{20} \approx (\delta\rho_c Y_e/5.86\ 10^{10}\ {\rm g\ cm^{-3}})^{1/3}$, and that the radius of the core is given by $R = (3\ M_{cc}/4\pi\delta\rho_c)^{1/3}$. The condition for trapping ($t_{\rm diff} > t_{cm}$) can thus be written as a condition on the central density: $\rho_c > \rho_{tr}$ where the trapping density is given by

$$\rho_{tr} = \rho_0\left[\frac{\chi/\delta}{X_{\rm H}A_{\rm H}/60 + 0.2X_n}\left(\frac{M_\odot}{M_{cc}Y_e}\right)^{2/3}\left(\frac{5.3\ 10^{10}}{\rho_0}\right)^{3/2}\right]^{1/(1-\beta)} \quad (18)$$

Since the characteristic density must lie between the central density and the core-mantle density, the value of δ should be in the range $1 > \delta > 0.02$.

During our computations we have monitored the ratio $t_{\rm diff}/t_{cm}$. The curves in Figs. 1–4 are shown dashed where trapping is possible (for $\delta = 1$ and $M_{cc} = 0.5\ M_\odot$) and the trapping densities are listed in Table 1. For rapidly evolving cores ($\chi = 2$) trapping can occur very early (i.e. for Y_e above 0.44 where the nuclear properties are fairly well known). For slower, more realistic contractions ($\chi = 10, 30$) the neutrions may still be escaping freely when $Y_e \lesssim 0.33$, and dynamical collapse begins [Eq. (7)]. In these cases the lepton number in the core-center may be very low when neutrino trapping finally occurs.

It is possible that neutrino trapping can be important even if $t_{\rm diff} < t_{cm}$. If the electron capture rate exceeds the neutrino diffusion rate, the density of neutrinos in the core would increase. The accumulated neutrinos would tend to block further electron captures resulting in electron capture rates which are below the values we have used which do not include the neutrino inhibition factors. In our calculations, however, the electron capture rate does not exceed the diffusion rate until after $t_{cm} < t_{\rm diff}$, so that Eq. (18) is the appropriate expression for the density at the onset of neutrino trapping.

The central density at the onset of dynamic collapse ρ_{dc} can be roughly estimated from Figs. 1–3 and Eq. (7). For central core masses M_{cc} between 0.3 $M_\odot$ and 0.7 $M_\odot$ and for values of χ of 10 and 30, our calculated values of ρ_{dc} lie in the range $0.3 - 1\ 10^{11}\ {\rm g\ cm^{-3}}$. More exact determinations are shown in Table 1. For $\chi = 2$ trapping of neutrinos occurs before dynamic collapse begins, and our calculations therefore become invalid before ρ_{dc} is reached.

Now consider the influence of changing the nuclear parameters on the determination of ρ_{dc}. In the formulae given by Epstein and Arnett (1975) (EA) the rate of electron captures and the neutrino emissivity are determined for $Y_e \geqq 0.44$. To extend our computations up to the onset of dynamic collapse we have extrapolated these results to values of $Y_e < 0.44$ as described in the Appendix. In Table 1 we show how the uncertainties in these extrapolations (as estimated in the Appendix) affect our determination of ρ_{dc}, ρ_{tr}, E_ν and ε_{20}. Here we have illustrated the influence of the parameters a and d. During core collapse, ρ and T remain in the region for which most of the matter is bound in heavy nuclei but the few free protons dominate the electron capture rate. (Region C of EA Fig. 2.) In this range the parameter a determines the electron capture rate and parameter d determines the mean neutrino energy. Variations of the other important parameters produce results which are intermediate to the extreme values of ρ_{dc}, E_ν and ε_{20} shown in Table 1.

III. Explosions by Neutrino Momentum Deposition

By dint of the elastic scattering of neutrinos from nuclei a flux of neutrinos exerts an acceleration a_ν on each element of matter through which it passes.

$$a_\nu = \frac{\kappa H}{c}. \quad (19)$$

Table 1. Central density at the onset of dynamic collapse, trapping density, total energy radiated in neutrinos prior to trapping and mean energy of neutrinos when trapping occurs

Parameter[a]	$\rho_{dc}/10^{11}$ g cm^{-3}	$\rho_{tr}/10^{11}$ g cm^{-3}	$E_\nu/10^{51}$ erg	ε_{20}[e]
$\chi = 2$	6.3[b]	0.6	0.5	0.8
$\chi = 30$	0.3	>0.8[d]	>3.3[d]	0.6[c]
$\log a = -3.5$	10.0[b]	1.3	0.9	1.0
$\log a = -1.5$	0.3	>0.5[d]	>3.3[d]	0.5[c]
$d = 0.9$	0.8	2.6	2.3	0.9
$d = 1.8$	0.7	1.1	2.5	0.8
$M_{cc} = 0.3\ M_\odot$	0.9	1.5	2.9	0.8
$M_{cc} = 0.7\ M_\odot$	0.5	1.2	2.5	0.8

[a] The parameters are set at $\chi = 10$, $\log a = -2.51$, $d = 1.52$, $M_{cc} = 0.5\ M_\odot$ and $\rho_0 = 10^9$ g cm^{-3}, except for the changes which are noted here
[b] Very uncertain, because trapping occurs before ρ_{dc} is reached
[c] Uncertain, because trapping occurs for $Y_e \lesssim 0.15$
[d] This value is at $Y_e = 0.15$ and represents a lower limit since trapping occurs for $Y_e \lesssim 0.15$
[e] ε_{20} is the mean neutrino energy in units of 20 MeV

Here κ is the specific opacity [$\kappa = 1/\lambda\rho$, see Eq. (17)], and H is the power per unit area of the neutrino radiation. To determine how much energy is transmitted to an element of matter by the neutrino flux it is necessary to consider the other forces acting on the matter and to distinguish between prompt and slow neutrino emission and between the inner and outer regions of the core (the core-center and the core-mantle).

Prior to the onset of dynamic collapse the entire core is approximately in hydrostatic equilibrium, so that the net acceleration on every element of the core is much less than the acceleration of gravity.

The core-center goes into dynamic collapse and within a short time (of the order of t_{cc}) after the onset of this collapse a large flux of neutrinos is emitted. Since the mass distribution of the outer regions of the core can change only slightly during this short interval, there remains a region beyond a radius r_m where the matter remains in approximate hydrostatic balance except for the outward acceleration due to the neutrinos (r_m defines the inner boundary of what we have been calling the core-mantle). Within r_m (i.e. in the core-center) the electron pressure of the matter is decreased due to electron captures so that it does not compensate the gravitational forces. The matter is thus subject to an inward acceleration of the order of the gravitational acceleration and an outward acceleration due to the neutrino flux. A net outward acceleration can be achieved in this region if the neutrino acceleration [Eq. (19)] is greater than or of the order of the acceleration of gravity. This occurs only if the neutrino luminosity exceeds the "Neutrino Eddington Luminosity" (Schramm, 1976). It has been found in detailed calculations that this condition is not fulfilled, especially during the initial emission of neutrinos (Wilson et al., 1975; Wilson, 1976; Bruenn et al., 1977). It therefore seems unlikely that the material in the core-center can acquire momentum or energy from the neutrino flux (except if the matter were already being ejected with high velocities due to hydrodynamical effects).

At the very center thermal pressure and nuclear forces come into play, but the matter in this region is trapped in a very deep gravitational well and cannot be significantly accelerated by the neutrinos.

In the core-mantle beyond r_m the pressure gradients and the gravitational force are nearly balanced so that the matter can acquire outward momentum from the neutrinos even if the luminosity is significantly below the Eddington limit.

An element of matter at radius r is exposed to an outward energy flux of

$$H = \frac{L_\nu}{4\pi r^2}, \tag{20}$$

where L_ν is the total neutrino luminosity. If $r > r_m$, the matter can attain an outward velocity

$$v = \int_{\Delta t} a_\nu dt = \frac{\kappa}{4\pi c r^2} E_\nu, \tag{21}$$

where

$$E_\nu = \int_{\Delta t} L_\nu dt \tag{22}$$

is the total energy in neutrino radiation during the short interval Δt.

The energy which the core-mantle can gain due to this burst is

$$E_{cm} = \int_{r_m}^{\infty} 2\pi\rho v^2 r^2 dr \tag{23}$$

$$= \int_{r_m}^{\infty} \frac{\kappa^2 E_\nu^2 \rho dr}{8\pi c^2 r^2}. \tag{24}$$

Taking the κ to be a constant through the region $r > r_m$ (this is the case if the composition and neutrino spectrum does not change with radius) and assuming that the density is decreasing as a power law; i.e. $\rho = \rho_m(r/r_m)^{-\alpha}$ gives

$$E_{cm} = \frac{E_\nu^2 \kappa^2 \rho_m}{8\pi(\alpha + 1) r_m c^2}. \tag{25}$$

This expression is valid if the neutrino burst is sufficiently short so that the work done by other forces during this interval is small compared to E_{cm} and if the velocities remain non-relativistic.

Equation (25) can be cast in a more intuitive form if we make the following suggestive substitutions: one neutrino mean free path is $\lambda = 1/\kappa\rho$, a "characteristic" optical depth for neutrinos in the core-mantle is $\tau_{cm} = r_m/\lambda$, the mass within one mean free path is approximately $M_\lambda = 4\pi r_m^2 \lambda \rho_m$ and the total momentum of the neutrino flux is $P_\nu = E_\nu/c$. In terms of these variables the energy transferred to the mantle is

$$E_{cm} = \frac{P_\nu^2}{2M_\lambda} \frac{\tau_{cm}}{(\alpha + 1)} \tag{26}$$

From this one can see that the efficiency with which energy is imparted to the core-mantle is necessarily small if (1) the minimum mass in which the neutrinos can be stopped is large ($M_\lambda \gg P_\nu/c$) or if (2) the core-mantle is optically thin to neutrinos ($\tau_{cm} \ll 1$) so that the neutrinos escape with nearly all of their initial energy.

Equation (25) shows that energy is imparted to the core-mantle most effectively when the core-mantle is such that ρ_m/r_m is large. Since the core-mantle structure is related to the central density at the onset of dynamical collapse, we define

$$\rho_m = \beta_1 \rho_{dc}, \tag{27}$$

$$\frac{4\pi}{3} r_m^3 \beta_2 \rho_{dc} \equiv M_{cc}. \tag{28}$$

To estimate β_1 and β_2 we recall that the stellar core is similar to an $n = 3$ polytrope prior to the onset of dynamic collapse and thus has a mean density of about 0.02 of the central density. Because the inner boundary of the core-mantle will be fairly deep within the core, we set $\beta_1 = \beta_2 = 0.1\phi$ where ϕ is presumably of order unity. With these relations and with Eq. (17), Eq. (25) can be written

$$E_{cm} = 1.5\ 10^{50} \left(\frac{E_\nu}{5\ 10^{51}\ \mathrm{erg}}\right)^2 \frac{\varepsilon_{20}^4 \phi^{4/3}}{(\alpha + 1)} \left(\frac{0.5\ M_\odot}{M_{cc}}\right)^{1/3} \times \left(\frac{\rho_{dc}}{10^{11}\ \mathrm{g\ cm^{-3}}}\right)^{4/3} \mathrm{erg} \tag{29}$$

Here we have used the same notation as in Eq. (18) and have set $X_{\mathrm{H}} = 1$ and $A_{\mathrm{H}} = 60$.

The quantities in the above expression have been normalized to correspond to recent estimates. Detailed calculations (Wilson, 1976; Mazurek, 1977a,b; Chechetikin et al., 1977) and approximate analytic treatments (Bruenn et al., 1977) have shown that approximately 5 10^{51} erg of neutrinos with energies in the neighborhood of 20 MeV escape from the core in the early phase of dynamic collapse. The possible values of E_ν and ε_{20} for the neutrinos which are emitted during the initial collapse are shown in Table 1. Since additional neutrinos may be copiously emitted later after the core-center has rebounded at high densities and re-expanded, the above estimates are consistent with our calculations. A value of ϕ of order unity also appears to be consistent with numerical studies (Wilson, 1976; Arnett, 1977b; Chechetikin et al., 1977), though this must be tested by additional calculations.

The observed properties of Type II supernovae can be explained if the values of E_{cm} are in the range $3 - 10\ 10^{50}$ erg (Falk and Arnett, 1977). From equation (29) it is apparent that core-mantle energies of this magnitude are realizable for plausible values of ρ_{dc}, E_ν and ε_{20}.

IV. Summary

The core of a pre-supernova star is described as being composed of two regions, the core-center and the core-mantle. A supernova event occurs when the core-center undergoes violent dynamic collapse and imparts a small fraction of the released gravitational energy to the material of the core-mantle.

The core evolves to higher densities and pressures at a relatively slow steady rate until the onset of dynamic collapse is reached. We estimate that dynamic collapse of the core-center begins when small changes in the core-mantle induce large responses in the core-center. It is found that this occurs when $Y_e \approx 0.3$–0.4 [see Eq. (7) for a more precise criterion].

The factors which affect the core evolution up until the onset of dynamic collapse have been explored. The influence of the stellar structure models is allowed for by parametrizing the rate at which the core is hydrostatically compressed and the mass of the core-center region. The equation of state and the rate of the electron capture reactions are parametrized using the formulae of EA.

Several conclusions can be drawn from these calculations. (1) From the end of core silicon burning ($T \approx 4\ 10^9$ K) and prior to neutrino trapping or dynamic collapse, the core-center tends to evolve to a common $\rho - T$ trajectory regardless of the initial density. This result is only weakly dependent on the compression rate, the uncertainties in the equation of state, the beta rates and the initial value of Y_e within the range $0.5 \lesssim Y_e \lesssim 0.4$. (2) The start of neutrino trapping in the core (defined as when the time for the neutrinos to diffuse out of the core first becomes long relative to the compression timescale) depends sensitively on the compression rate and the uncertainties in the properties of neutron rich matter. The realistic uncertainties in the problem are sufficiently great that one cannot yet say whether any significant trapping of neutrinos occurs: the value of Y_e when trapping begins could be greater than 0.46 or less than 0.15 (also see Arnett, 1977b; Mazurek, 1977b; Sato, 1975). (3) The central density at the onset of the dynamic collapse is similarly uncertain (see Table 1). Plausible values can lie in the range $3\ 10^{10} \lesssim \rho_{dc} \lesssim 10^{12}$ g cm^{-3}.

During the dynamic collapse of the core-center and the early or prompt emission of neutrinos from this region, the core-mantle is for the most part supported by normal pressure stresses. The outstreaming neutrinos provide an additional outward thrust to the core-mantle material. The estimates presented in Sect. III show that (1) the energy imparted to the core-mantle, E_{cm}, depends on the total flux of prompt neutrinos E_ν and on the neutrino energy spectrum (i.e. on ε_ν) rather than on the ratio of the neutrino luminosity to the "Neutrino Eddington Luminosity". (2) The value of E_{cm} depends on the structure of the core-mantle and hence on the value of ρ_{dc}; $E_{cm} \sim \rho_{dc}^{4/3}$. (3) The plausible values for ρ_{dc}, E_ν and ε_ν span the range from values which would yield essentially no supernova display to values which should correspond to energetic supernova events.

Ideally all of the above claims should be tested by detailed numerical simulations. In such calculations special care must be given to insuring that the initial models are hydrostatic and stable so that a quasi-hydrostatic core-mantle can develop. Also, the region from r_m (where the material pressure gradient nearly offsets gravity) to where the core-mantle is highly transparent to neutrinos must be finely zoned to permit a realistic representation of the complex radiative-hydrodynamic phenomena which should occur in this region.

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Appendix

Equation of State and Neutrino Emissivity

To study the evolution of a stellar core *prior* to the onset of dynamical collapse one needs to know the internal energy, pressure, rate of neutronization and neutrino emissivity for matter at densities below several times 10^{12} g cm^{-3} and temperatures between $4\ 10^9$ K and $5\ 10^{10}$ K. In the relevant regime (see Figs. 1–4) the nuclei obtain full nuclear statistical equilibrium on very short timescales relative to any dynamical timescale of the problem, the electrons are partially degenerate and strongly, but not fully, relativistic, and the plasma is dense in the sense that the Coulomb pressure is a few percent of the total pressure and the binding energy of heavy nuclei are altered by about 10% by electrostatic interaction with the plasma.

The internal energy and the pressure can be separated into the contributions due to the free ions, the electrons, the nuclear binding of the nuclei and the Coulomb binding of the plasma, as shown in Eqs. (14) and (15).

The ion pressure and energy are given by

$$P_I = Y_I \rho R T, \tag{A1}$$

$$u_I = 3/2 P_I/\rho, \tag{A2}$$

where R is the gas constant and Y_I is the total number of nuclei per atomic mass unit. The number density of nucleus j divided by the baryon density is denoted by Y_j and

$$Y_I = \sum_j Y_j. \tag{A3}$$

The nuclear energy is

$$u_{\rm nuc} = -\sum_j Y_j A_j B_j - (m_n - m_p) Y_e, \tag{A4}$$

where B_j is the binding energy per nucleon and A_j is the nuclear mass. The Coulomb pressure and energy are approxi-

Table A1. Fitting parameters

	B_H	A_H	log a	b	d	e	f
high	8.60	50.0	−1.50	1.00	1.80	2.30	1.20
medium	8.78	62.2	−2.51	2.16	1.52	1.49	0.85
low	8.90	70.0	−3.50	2.50	0.90	1.00	0.40

	log c									
$T/10^9$	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5	15.0
high	0.50	0.55	0.65	0.70	0.80	1.00	1.10	1.20	1.25	1.30
medium	0.43	0.45	0.51	0.55	0.60	0.68	0.76	0.86	0.95	1.01
low	0.10	0.15	0.20	0.25	0.40	0.50	0.60	0.70	0.75	0.80

mated by

$$P_{\rm coul} = -\frac{3}{10}\left(\frac{4\pi}{3}\right)^{1/3} e^2(\rho N)^{4/3} Y_e^{1/3} \sum_j Z_j^{5/3} Y_j, \tag{A5}$$

$$u_{\rm coul} = 3P_{\rm coul}/\rho, \tag{A6}$$

where N is Avogadro number and Z_j is the nuclear charge. Equation (A1)–(A6) are evaluated with the results of EA with certain modifications which are discussed below.

In the temperature density regime of interest here the electron pressure and energy are approximated by

$$P_e = P_0 + P_t\left[1 + \left(\frac{24P_0}{\pi^2 P_t}\right)^2\right]^{-1/2}, \tag{A7}$$

$$U_e = \frac{3P_e}{\rho}\left(1 - \frac{4}{3\chi}\right), \tag{A8}$$

where

$$P_0 = 1/8\left(\frac{1}{\pi}\right)^{1/3} hcn_e^{4/3} = 1.244\ 10^{15}\,(\rho Y_e)^{4/3}, \tag{A9}$$

$$P_t = n_e kT = 8.3143\ 10^7\,\rho Y_e T, \tag{A10}$$

$$\chi = \frac{4}{3}\left(\frac{8\pi}{3} n_e\right)^{1/3}\frac{mc}{h} = 1.009\ 10^{-2}\,(\rho Y_e)^{1/3}. \tag{A11}$$

The symbols have the usual meanings and cgs units are used. These expressions are strictly valid for small T and large ρ and are good to better than 3% through out the range of the present calculations.

The terms in Eqs. (4)–(7) and (A1)–(A6) were evaluated by modifying the results of EA. As pointed out by Mackie (1976) the binding energy of the nuclei are changed by the interaction with the plasma. In the Wigner-Seitz approximation the binding energies per nucleon of nucleus i becomes

$$B_i' = B_i + \frac{9}{10}\left(\frac{4\pi}{3}\right)^{1/3}\frac{(Z_i^{5/3} - Z_i)}{A_i} e^2 n_e^{1/3}. \tag{A12}$$

This expression has been used to improve the binding energies B_α and B_H in EA.

Some of the sums over nuclear species are explicitly calculated in EA. Others, for instance Eq. (A5), are not. These latter sums are evaluated by using the 4 particle model for the nuclear constituents, neutrons, protons, alpha particles and a characteristic "heavy particle" of mass A_H and charge Z_H, as described in EA.

The results of EA are valid for $0.5 > Y_e > 0.44$. To extend these calculations to lower values of Y_e and to test the sensitivity of the present calculations on these extensions we have used the following procedure.

The parameters B_H, A_H and a through f are tabulated in Epstein and Arnett for $0.50 < Y_e < 0.44$. From the variation in these parameters in this range we obtain an estimate of how they might vary in the poorly understood range below $Y_e < 0.44$. On this basis three estimates were made for each of these parameters as shown in Table A1. The medium value is the value of the parameter at $Y_e = 0.44$. The parameters were assumed to linearly evolve to one of these three values at $Y_e = 0.4$ and to remain constant thereafter.

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Note added in proof: Work of Bethe et al. (1978) indicates that electron capture reactions are supressed at high degrees of neutronization thereby preventing Y_e from attaining very low values. The other conclusions of this paper remain unaltered.